Exam. Code : 211003
Subject Code : 3852

M.Sc. Mathematics 3rd Semester

STATISTICS—I

Paper—MATH-577

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting **TWO** question from each Unit. All questions carry equal marks.

UNIT—I

I. (a) Describe Skewness and Kurtosis of the distribution. Also give their measures.

(b) In a frequency distribution, the coefficient of Skewness based upon the quartiles is 0.6. If the sum of the upper and lower quartiles is 100 and median is 38, find the value of upper and lower quartiles.

II. (a) For a random variable x, moments of all order exist. Denoting by μ_r, the r^{th} central moment of frequency distribution of x, show that :

$$\mu_{2r+1}^2 \leq \mu_{2r} \, \mu_{2r+2} .$$

(b) Show that in a discrete series if deviations (x) are small compared with mean M so that $(x/M)^3$ and higher power of (x/M) are neglected, we have

$$G = M\left(1 - \frac{1}{2}\frac{\sigma^2}{M^2}\right),$$ where G is geometric mean and σ^2 is variance of the series.

III. (a) For n events $A_1, A_2, \ldots, A_n$ belonging to sample space, show that :

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$

(b) The sum of two non-negative quantities is equal to 2n. Find the probability that their product is not less than $\frac{3}{4}$ times their greatest product.

IV. (a) Prove or disprove the following :

(i) If $P(A/B) \geq P(A)$, then $P(B/A) \geq P(B)$

(ii) If $P(A) = P(B) = p$, then $P(A \cap B) \leq p^2$

(iii) If $P(B/\bar{A}) = P(B/A)$, then A and B are independent.

(b) For any two events A and B, suppose that $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = p$.

For what value of p :

(i) A and B are mutually exclusive events.

(ii) A and B are independent events ?

UNIT—II

V. (a) Distinguish between a discrete and a continuous random variable. Also explain the concept of independence of random variables.

(b) Suppose that the random variable X takes values 1, 2, 3, ... with $P(X = j) = \frac{1}{2^j}$; j = 1, 1, 2, 3, ... respectively. Compute :

(i) P (X is even)

(ii) P (X is divisible by 3).

VI. (a) What do you mean by n-dimensional random variable ? Describe its probability function.

(b) Two dimensional random variable (X, Y) have the joint probability density :

$$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal and conditional distributions.

VII. (a) Suppose that the dimensions, X and Y, of a rectangular metal plate may be considered to be independent continuous random variables with following p.d.f.'s :

$$X : g(x) = \begin{cases} x-1, & 1<x\leq 2 \\ -x+3, & 2<x<3 \\ 0, & \text{otherwise} \end{cases}$$

$$Y ; h(y) = \begin{cases} \frac{1}{2}, & 2<y<4 \\ 0, & \text{otherwise} \end{cases}$$

Find the p.d.f. of the area of plate A = XY.

(b) The diameter of an electric cable, say X is assumed to be continuous random variable with p.d.f.

$f(x) = 6x(1 - x), 0 \leq x < 1.$

(i) Check that the above is a p.d.f.

(ii) Compute $P\left[X<\frac{1}{2}\middle|\frac{1}{3}<X<\frac{2}{3}\right]$.

VIII. (a) Define cumulative distribution function of the random variable and describe its properties.

(b) The two dimensional random variable (X, Y) has p.d.f.

$$f(x,y)=\begin{cases}\binom{y}{x}p^{x}(1-p)^{y-x}\dfrac{e^{-\lambda}\lambda^{y}}{y!}, & x=0,1,2,\ldots\ldots,y;\ y=0,1,2,3,\ldots\ldots\\ 0 & \text{otherwise}\end{cases}$$

Find the marginal distributions of X and Y. Also examine whether the random variable X and Y are independent.

UNIT—III

IX. State and prove Lindeberg-Levy central limit theorem.

X. (a) Define moment generating function. State and prove its important properties.

(b) Let X and Y be jointly distributed with p.d.f.

$$f(x,y)=\begin{cases}2, & 0<y<x<1\\ 0, & \text{otherwise.}\end{cases}$$

Find the conditional expectation E(X/Y = y).

XI. (a) If random variable X has p.d.f.

$f(x)=\dfrac{1}{2\theta}e^{-\frac{|x-\theta|}{\theta}}, -0<x<\infty, \theta>0,$ then find the moment generating function of X. Hence find the value of E(X) and V(X).

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

XII. (a) (i) Let X be a non-negative continuous random variable with cumulative distribution function F. Show that :

$$E(X) = \int_0^\infty (1 - F(x))\, dx .$$

(ii) If the possible values of a random variable are 0, 1, 2, ..., then show that :

$$E(X) = \sum_{n=0}^{\infty} P[X > n] .$$

(b) State and prove Chebyshev inequality.

UNIT—IV

XIII. (a) Obtain mean deviation from mean for normal distribution.

(b) Define hypergeometric distribution and find its mean.

XIV. (a) Show that the exponential distribution 'lacks memory'.

(b) Show that the sum of independent Gamma variables is also a Gamma variable.

XV. (a) If $X_1, X_2, ..., X_n$ are independent randoms variables, each X_i having an exponential distribution with parameter θ_i; i = 1, 2, ..., n, then find the distribution of $z = \min(X_1, X_2, X_3, ..., X_n)$.

(b) If the random variables $X_1, X_2, ..., X_k$ have a multinomial distribution, show that the marginal distribution of X_i is a binomial distribution with parameters n and p_i, i = 1, 2, ..., k.

XVI. (a) X is a Poisson variable with mean λ. Show that $E(X^2) = \lambda E(X + 1)$. If λ = 1, show that :

$$E|X-1| = \frac{2}{e}.$$

(b) Let X and Y be independent binomial variables, each with parameters n and p. Find P(X = Y).

UNIT—V

XVII. (a) Find the correlation coefficient between $lx + my$ and X + Y, when correlation coefficient between X and Y is ρ.

(b) Describe Principle of least square. Using this obtain a line of regression of Y on X for the case of bivariate data.

XVIII. (a) X and Y are two random variables with respective variances σ_X^2 and σ_Y^2, having the correlation coefficient ρ between them. If U = X + KY and $V = X + (\sigma_X/\sigma_Y)Y$, find the value of K so that U and V are uncorrelated.

(b) Differentiate between partial correlation and multiple correlation for the case of multivariate distribution.

XIX. (a) Define correlation ratio η_{XY} and prove that :

$$\rho^2 \leq \eta_{XY}^2 \leq 1$$

where ρ is the correlation coefficient between the X and Y.

(b) In usual notations, prove that :

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2} \leq r_{12}^2.$$

XX. (a) When are two attributes said to be (i) positively associated and (ii) negatively associated ? Also define complete association and dissociation of two attributes.

(b) The following table is reproduced from a memoir written by Karl Pearson :

		Eye Colour in Son	
		Not Light	**Light**
Eye Colour in Father	**Not Light**	230	148
	Light	151	471

Discuss if the colour of son's eyes is associated with that of father.